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Review of drag coefficients on gas – liquid tower: the drag coefficient independent and dependent on bubble diameter in bubble column experiment

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ABSTRACT

Drag coefficient independent on bubble diameter is required to ease design sieve trays or bubble column, through simulation of computational fluid dynamic. In this paper, the drag coefficients, independent or dependent on the diameter, are reviewed for gas-liquid system. A number of drag coefficients are used for Computational Fluid Dynamics (CFD). Different forces are entered to the liquid-bubble separation surface in diverse directions. Forces are investigated with mathematical proving for Newtonian fluids and Eulerian coordinate. Finally, the external force as a new force, enter to the drag coefficient equations. Drag coefficient is included force coefficient. Drag force is entered in momentum equations. Drag coefficient is used in two-phase systems which bubbles and liquid are activated as dispersed and continuous phase, respectively. Bubbles and liquid are in contact with each other in separation surface on bubble. Drag force is created slip on separation surface. The drag coefficients are investigated depended on the size and configuration of bubbles. The drag coefficient of Krishna et al is used dependent on bubble diameter. Schiller - Nauman model drag coefficient is estimated with 9% error and dependence on bubble diameter. In this article, the modern drag coefficients are studied independent on the diameter and shape of the bubble. The Drag coefficients are resulted theoretical, mathematical and experimental independent and dependent of diameter bubble. The new Drag coefficient is presented dependent on surface tension and diameter of the tower hole with 6.3 of error approximately.

1. Introduction

Towers are applied with gas - liquid system in large part of industry [1]. Momentum equations are governed towers. Bubble columns are designed in biochemical processes, such as fermentation, biological water purification and fuel cells production by synthetic gas conversion processes and chemical processes such as polymerization, chlorination and oxidation [1]. Bubbles are distributed in liquid phase in the gas phase bubble columns, from the bottom of the tower [1]. Shapes of bubbles are varied depending on different velocities and liquid-gas flow regimes.

Various forms of bubbles are developed such as spherical, ellipsoid and cap bubble [9]. Researchers are received main problem in liquid-gas system [9]. Important problem is analyzed drag coefficients independent on the diameter and bubble shape [23].

The drag coefficient is obtained with study of the order of magnitude in mathematical equations for different forces and direction absolute value the separation surface as well as solving governing mathematical equations [23].

Overall purposes of present paper are as follows:

- The drag coefficients are presented various forces evaluation with excellent precision.
- The drag coefficients are obtained independent on the diameter and bubble shape. Equal equations are offered to the bubbles diameter really. There are replaced to equal diameter.

2. Previous works

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The drag coefficient of the two-phase fluid is obtained by several researchers (Allen et al 1900, Langmuir et al 1948, Dalle Ville et al 1948, Gilbert et al 1955, Moore 1963, Kurten et al 1966) [2, 3, 4, 5, 6, 7]. (Abraham et al 1970, Clift et al 1970, Tanaka et al 1970, Ihme et al 1972, Brauer et al 1972, White 1974, Ma and Ahmadi 1990, Grevskott et al 1996, Tsuchya et al 1997, Lane et al 2000, Tomiyama 1998, 2004, Hameed et al 2015) [8,9,10,11,12,13,14,15,16,17,18,19,20].

Schiller and Newman (1935) [21] provided drag coefficient by dividing the bubbles shapes in different streams after precise measurements of bubble-raising speed. The drag coefficients are obtained from suitable advanced equation dispersed flow for bubble forms. Figurations are consisted spherical, ellipsoid and condensed cap particles. All drag coefficients depend on the bubble diameter. In this model, Drag coefficient is showed constant value for spherical forms.

Ishii and Zuber (1979) [22] Suggested a correlation coefficient for different bubble structures in a wide range of Reynolds numbers. The Drag coefficient provided depending on the bubble diameter. The drag coefficient is shown value constant for spherical shapes.

Krishna et al. (1999) [23] proposed a drag coefficient for bubble movement in gas-liquid towers, in Euler-Euler coordinate based on the slip and gas hold up for hydrodynamic sieve trays. The drag coefficient is presented depended on the diameter, but in the momentum equation, vanished using the Bennett et al (1983) [24] relationship.

Noriler et al. (2008) [25] showed correlation coefficient for the towers in the Euler- Euler structure and dependent on bubble diameter in gas - liquid system.

Zhang et al. (2008) [26] provided drag coefficient based on proof mathematical equations, for direct movement of the bubble in the static fluid via the balance of forces interacting on the bubble.

3. Mathematical model

The model is consisted gas-liquid system in the Euler - Euler structure. The gas-liquid phases are interacted together possess separate continuity and momentum equations. The continuity and Navier - Stokes equations are written with average Reynolds for gas and liquid phase as shown in 1- 4 relations.

Gas phase:

$$\frac{\partial}{\partial t}(\varepsilon_g \rho_g) + \nabla.(\varepsilon_g \rho_g u_g) = 0 \qquad 1$$

Liquid phase:

$$\frac{\partial}{\partial t}(\varepsilon_l \rho_l) + \nabla.(\varepsilon_l \rho_l u_l) = 0 \qquad 2$$

Total volume of gas and liquid is obtained as:

$$\varepsilon_g + \varepsilon_l = 1 \qquad 3$$

$\rho_l, \rho_g, u_l, u_g, \varepsilon_g$ and ε_l are liquid-phase density, gas-phase density, liquid velocity, gas velocity, gas-phase fraction and liquid-phase fraction, respectively.

Momentum equations:

Balance of momentum is obtained with the accumulated and momentum flux composition of the molecular and convective momentum flux, Pressure, drag and other forces.

For gas-phase, momentum equation is:

$$\frac{\partial}{\partial t}(\varepsilon_g \rho_g u_g) + \nabla.(\varepsilon_g \rho_g u_g u_g) = -\varepsilon_g \nabla p_g + \nabla.(\varepsilon_l \mu_l^{eff} (\nabla u_l + \nabla u_l^T)) - M_{g,l} + \rho_l \varepsilon_l g \qquad 4$$

And for liquid-phase:

$$\frac{\partial}{\partial t}(\varepsilon_l \rho_l u_l) + \nabla.(\varepsilon_l \rho_l u_l u_l) = -\varepsilon_i \nabla p_i + \nabla.(\varepsilon_l \mu_l^{eff} (\nabla u_l + \nabla u_l^T)) + M_{g,l} + \rho_l \varepsilon_l g \qquad 5$$

μ_K and g, respectively, show molecular viscosity and gravity vector of k-phase. p_k is pressure field and has same value for gas and liquid phases; $p_i = p_g$. $M_{g,l}$ imply momentum transfer between the gas and liquid phases. In addition, the momentum flux is caused from speed fluctuations and turbulence which there are incorporated in diffusion.

3.1. Drag force

The drag force is slip on separation surface, one of the factors affecting momentum transfer. The drag force per unit volume is:

$$M_{g,l} = \frac{3}{4} \frac{\varepsilon_g \rho_l}{d_b} C_D |u_g - u_l| (u_g - u_l) \qquad 6$$

Where d_b, C_D and $M_{g,l}$ are bubble diameter, drag coefficient a relationship entering in momentum equations and the drag force per volume unit, respectively.

Now, applicate relations for simulation are described.

According to Schiller and Neuman approach in 1935, Drag coefficient obtained pursuant to the result. This method employed was also employed for gas-liquid, liquid - liquid as well as solid - liquid systems.

For condensed spherical particle:

In the dense spherical particles for Reynolds:

$$C_D(sphere) = \frac{24}{\mathrm{Re}_m}(1 + 0.15\,\mathrm{Re}_m^{0.687}) \qquad 7$$

$0 \le \mathrm{Re} < 1000$

Re_m is mixture Reynolds number:

$$\mathrm{Re}_m = \frac{\rho_l \left|\overrightarrow{U_g} - \overrightarrow{U_l}\right| D_b}{\mu_m} \qquad 8$$

To measure mixture viscosity, μ_m:

$$\mu_m = \mu_l (1 - \frac{\alpha_g}{\alpha_{max}})^{-2.5\alpha_{max}(\mu_g + 0.4\mu_l)/(\mu_g+\mu_l)} \quad 9$$

α_{max} is maximum amount of mixture and equals to 0.52.

Ellipsoid particle region are condensed for Reynolds above 1000:

$$C_D(ellipse) = \frac{2}{3}\sqrt{EoE} \quad 10$$

$$E = \frac{(1+17.67f^{6/7})}{18.67f} \quad 11$$

For condensed Spherical Cap Regime for: Reynolds above 1000;

$$C_D(cap) = \frac{8}{3}E' \quad 12$$

$$E' = (1-\varepsilon_g)^2 \quad 13$$

Dalla Ville in 1948 provided a relationship used for computational fluid dynamics in terms of Reynolds number.

While axial sliding speed depended on the bubble diameter.

$$C_D = (0.63 + \frac{4.8}{\sqrt{Re_l}})^2 \quad 14$$

Reynolds is given as equation 14:

$$Re = \frac{\rho_l U_{Slip}\, d_p}{\mu} \quad 15$$

U_{Slip} is the slip velocity, ρ_l liquid density, μ dynamic viscosity [4, 42].

White in 1974 proposed a relation depending on bubble diameter in order to simulate for gas-liquid devices, this Computational Fluid Dynamics relation, is used [13, 41].

$$C_D = 0.44 + \frac{24}{Re} + \frac{6}{1+\sqrt{Re}} \quad 16$$

In 1976, Grace et al carried out dimensional analysis for individual bubbles going up in static fluid. Grace et al concluded dynamics have fully expressed in accordance with dimensionless sub-groups such as EO, Re and MO. Note that the Re shows inertial force per viscose ratio. EO is floating to the surface tension force and MO is group property of two Phases. σ is based on tension surface between the two phases [29, 42].

$$C_{Dgl} = \frac{4gd_b(\rho_l - \rho_g)}{3U_T^2\rho_l} \quad ; \quad ellipsoid$$

$$C_D = 24/Re_l \qquad Re_l < 1 \quad 17$$

$$C_D = 0.44 \qquad 0 < Re_l < 1000$$

The first relation is for ellipsoid bubble.

Dimensionless values combination is as follows:

$$Re^2 = \frac{4}{3C_D}\sqrt{\frac{E_0^3}{M_O}} \quad 18$$

A correlation distribution in regimes possessing various forms of Reynolds was offered by Ishii and Zuber (1979) [22].

$$C_{D,sphere} = \frac{24}{Re}(1+0.1Re^{0.75}) \quad 19$$

$$C_D = \frac{2}{3}Eo^{0.5} \quad 20$$

$$C_{D,cap} = \frac{8}{3} \quad 21$$

In 1999, the drag coefficient C_D for simulation of computational fluid dynamics and liquid-gas towers with Drag correlation was presented by Krishna et al [23],

$$C_D = \frac{4}{3}\frac{(\rho_l - \rho_g)gd_g}{\rho_l|u_g - u_l|^2} \quad 22$$

This equation was presented within the Euler - Euler framework and used to raise swarm bubbles in the turbulent region.

$$|u_g - u_l| = \frac{U_g}{f_g^{average}} \quad 23$$

In this equation, $|u_g - u_l|$ is relative speed between gas and liquid and can be estimated as a function of surface speed, $U_g = Q_G / A_B$, $f_g^{average}$ or average gas hold-up was obtained from the equation provided by Bennett et al [24].

$$f_\beta^{average} = 1 - \exp\left[-12.55(U_g\sqrt{\frac{\rho_g}{\rho_l - \rho_g}})^{0.91}\right] \quad 24$$

Correct replacement is applied in momentum transfer equation of separation surface, in proper form of CFD [23]:

$$M_{gl} = f_g(\rho_l - \rho_g)\left[\frac{1}{(U_g/f_g^{average})^2}\frac{1}{(1-f_g^{average})}\right] \quad 25$$

For the stock regime: [1]

$$Re_G = \frac{\rho_L U_G d_G}{\mu_L} \quad 26$$

$$C_D = \frac{24}{Re_G} \quad 27$$

In another form of CFD applications are assembled simplified equations of overall system in steady-state conditions. This problem is empirically calculated by combining the following equations; the average drag coefficient β and average drag force F_D is shown in the form of equations 28 and 29 due to its application in multi-phase and two-phase fluids. [32]

$$F_D = \beta(u_l - u_g) \tag{28}$$

$$\beta = \frac{3}{4}\varepsilon_g \frac{C_D}{d_b}\rho_l \left| u_l - u_g \right| \tag{29}$$

Table 1, Models of drag coefficient

Researchers	Drag coefficient
Allen et al (1900) [2, 40]	$a) C_D = 10\,\mathrm{Re}^{-1/2}$, $2 < \mathrm{Re} < 500$ $b) C_D = 30\,\mathrm{Re}^{-0.625}, 1 < \mathrm{Re} < 1000$
Schiller and Naumann (1935) [21]	$a)\ C_D(sphere) = \frac{24}{\mathrm{Re}_m}(1 + 0.15\,\mathrm{Re}_m^{0.687})$ $\mathrm{Re}_m = \frac{\rho_l \left\| \overrightarrow{U_g} - \overrightarrow{U_l} \right\| D_b}{\mu_m},\ \mu_m = \mu_l (1 - \frac{\alpha_g}{\alpha_{\max}})^{-2.5\alpha_{\max}(\mu_g + 0.4\mu_l)/(\mu_g + \mu_l)}$ $b)\ C_D(ellipse) = \frac{2}{3}\sqrt{EoE}\ ,\ E = \frac{(1 + 17.67 f^{6/7})}{18.67 f}$ $c)\ C_D(cap) = \frac{8}{3}E'\ ,\ E' = (1 - \alpha_g)^2$
Lungmuir et al (1948) [3, 40]	$C_D = \frac{24}{\mathrm{Re}}(1 + 0.197\,\mathrm{Re}^{0.63} + 2.6 \times 10^{-4}\,\mathrm{Re}^{1.38})$, $1 < \mathrm{Re} < 100$
Dalle ville (1948) [4, 42]	$C_D = (0.63 + \frac{4.8}{\sqrt{\mathrm{Re}_l}})^2$
Gilbert et al (1955) [5, 40]	$C_D = 0.48 + 25\,\mathrm{Re}^{-0.85}$, $0.2 < \mathrm{Re} < 2000$
Moore et al (1965) [6, 40]	$C_D = \frac{48}{\mathrm{Re}}\left[1 - \frac{2.21}{\mathrm{Re}^{1/2}} + O(\mathrm{Re}^{-5/6})\right]$
Kurten et al (1966) [7, 40]	$C_D = 0.28 + \frac{6}{\mathrm{Re}^{1/2}} + \frac{21}{\mathrm{Re}}$
Abraham et al (1970) [8, 40]	$C_D = 0.2924(1 + 9.06\,\mathrm{Re}^{-1/2})^2$, $\mathrm{Re} < 6000$
Clift et al (1970) [9, 40]	$C_D = \frac{24}{\mathrm{Re}}(1 + 0.15\,\mathrm{Re}^{0.687}) + 0.42/(1 + 4.25 \times 10^4\,\mathrm{Re}^{-1.16})$, $\mathrm{Re} < 3 \times 10^5$
Tanaka et al (1970) [10, 40]	$\log_{10} C_D = a_1 w^2 + a_2 w + a_3$, $w = \log_{10}\mathrm{Re}$, $\mathrm{Re} < 7 \times 10^4$
Ihme et al (1970) [11, 40]	$C_D = 0.36 + \frac{4.48}{\mathrm{Re}^{0.573}} + \frac{24}{\mathrm{Re}}$, $\mathrm{Re} < 10^4$
Brauer et al (1972) [12, 40]	$C_D = 0.4 + \frac{4}{\mathrm{Re}^{1/2}} + \frac{24}{\mathrm{Re}}$, $\mathrm{Re} < 3 \times 10^5$
White (1974) [13, 41]	$C_D = 0.44 + \frac{24}{\mathrm{Re}} + \frac{6}{1 + \sqrt{\mathrm{Re}}}$
Grace (1976) [29, 42]	$C_D = \frac{4}{3}\frac{(\rho_l - \rho_g) g d_b}{\rho_l U_T^2}$, *bubble is ellipsoid* $\begin{cases} 24/\mathrm{Re}\ ,\ \mathrm{Re}_l \le 1 & ,\ \textit{bubble is spherical} \\ 0.44\ ,\ 1 < \mathrm{Re}_l < 1000, & \textit{bubble is spherical} \end{cases}$
Ishii and Zuber (1979) [22]	$C_{D,sphere} = \frac{24}{\mathrm{Re}}(1 + 0.1\,\mathrm{Re}^{0.75})$ $C_D = \frac{2}{3}Eo^{0.5}$ $C_{D,cap} = \frac{8}{3}$
Ma and Ahmadi (1990) [14]	$C_D = 24(1 + 0.1\,\mathrm{Re}_l^{0.75})/\mathrm{Re}_l$
Gravskott (1996) [15]	$C_D = \frac{5.645}{Eo^{-1} + 2.835}$

Tsuchiya (1997) [16]	$C_D = (1-\varepsilon_g)^2 \ , \ \max \begin{cases} 24(1+0.15\,\mathrm{Re}_l^{0.687})/\mathrm{Re}_l, \\ \frac{8}{3}(\frac{Eo}{Eo+4}) \end{cases}$
Tomiyama (1998) [18]	$C_{D0} = \max\left\{ \min\left[\frac{16}{\mathrm{Re}}(1+0.15\,\mathrm{Re}^{0.687}), \frac{48}{\mathrm{Re}} \right], \frac{8}{3}\frac{E_0}{E_0+4} \right\}$
Krishna et al (1999) [23]	$C_D = \frac{4}{3}\frac{\rho_l-\rho_g}{\rho_l} g d_g \frac{1}{\lvert u_g - u_l \rvert^2} \ , \ \lvert u_g - u_l \rvert = \frac{U_g}{f_g^{average}} \ , \ U_g = {Q_g}/{A_B}$ $(Bennett\ et\ al\ 1983) \ , \ f_\beta^{average} = 1 - \exp\left[-12.55 (U_g \sqrt{\frac{\rho_g}{\rho_l - \rho_g}})^{0.91} \right]$
Lapple [33, 40]	$C_D = \frac{24}{\mathrm{Re}}(1+0.125\,\mathrm{Re}^{0.72}) \ , \ \mathrm{Re} < 1000$
Rumpf [34, 40]	$C_D = 2 + {24}/{\mathrm{Re}} \ , \ \mathrm{Re} < 10$ $C_D = 1 + {24}/{\mathrm{Re}} \ , \ \mathrm{Re} < 100$ $C_D = 0.5 + {24}/{\mathrm{Re}} \ , \ \mathrm{Re} < 10^5$
Tomiyama (2004) [19]	$C_D = \frac{8}{3}\frac{Eo(1-Eo^2)}{E^{2/3}Eo + 16(1-E^2)E^{4/3}} f(E)^{-2}$
Noriler et al (2008) [25]	$C_D = 0.28 + \frac{6}{\mathrm{Re}^{0.5}} + \frac{21}{\mathrm{Re}}$ $C_D = 0.2924(1+9.06\,\mathrm{Re}^{-0.5})^2$
Li Zhang (2008) [16]	$C_D = \frac{3}{4} d_e \frac{(\rho_l - \rho_g)g}{\rho_l U_T^2} \ , \ d_e = \sqrt[3]{\frac{6V}{\pi n}}$
Hameed et al (2015) [20]	$C_D = (\frac{32}{\mathrm{Re}})(\frac{(\alpha+1/2)}{(1-\alpha)^3}(\frac{\mu_0}{4\pi(\mu_0+\mu_i)}(2\beta + \sin(\beta) - \sin(2\beta)) - \frac{1}{3}\sin(3\beta) + \frac{\mu_0 + \frac{3}{2}\mu_i}{\mu_0+\mu_i}))(1+0.15\,\mathrm{Re}^{0.687})$

Mathematical relations 32 and 33 [34]:

$$\sum F = m a$$

Where;

$$F_w + F_{boy} + F_d + F_G + F_{cen} + F_{ext} + F_{am} = F_I \quad 33$$

Where F_I, F_{am}, F_{ext}, F_{cir}, F_G, F_d, F_{boy} and F_w are Inertial force, added mass force, external force, centrifugal force Center, gravity force, drag and buoyancy force bubbles weight.

$$m_l g - m_g g - \frac{1}{2} C_D \rho_l A U_T^2 + G(\frac{m_1 m_e}{r_{eq}^2}) + m_g a_{cen} + m^{\bullet} u - \varepsilon_1 \varepsilon_2 \rho_l C_V \frac{D(u_1 - u_2)}{Dt} = ma \quad 34$$

Added mass force for non-uniform flow interacts with bubble. Because of steady-state flow is entered and it is monotonic, the added mass force is ignored.

$$F_V = -\varepsilon_1 \varepsilon_2 \rho_l C_V \frac{D(u_1 - u_2)}{Dt} \quad 35$$

Gravity force emerges from interaction of the gravitational constant, the bubble mass, the equivalent bubble mass and equivalent distance.

$$F_G = G \frac{(m_1 m_{eq})}{r_{eq}^2} \quad 36$$

Equivalent bubble mass is analyzed equal to the total sum of available bubbles in the investigated tower. Equivalent distance is equal to total distance of all bubbles. Bubble mass is calculated by multiplying density in bubble volume.

$$m_1 = \rho_g V_{bub} \quad 37$$

If the bubble diameter is estimated as 0.000001- 0.1 m, bubble volume is estimated as 10^{-3}-$10^{-18}\,m^3$. Density is also in the order of one, multiplying mass order in the volume of the bubble the order of bubble mass is 10^{-3}-10^{-18} kg. If the bubbles number in the tower is in the amount maximum 10,000, the amount of equivalent mass via multiplying the bubbles number in order is equal to 10-10^{-14}kg. Equivalent distance of bubble with another bubble is in order of 1-1000 m. The gravitational constant is of the order of 10^{-11}. Placement of these orders in gravity force relationship, for maximum values of gravity force [39]:

$$F_G \approx 10^{-11}\frac{10^{-3}\times 10}{1000^2}\approx 10^{-8} \tag{38}$$

And for minimum values of gravity force:

$$F_G \approx 10^{-11}\frac{10^{-18}\times 10^{-14}}{(1)^2}\approx 10^{-43} \tag{39}$$

The order of gravity force is ignored in balance force equations.

The order of the centrifugal force is decreased in opposition to the surface tension force and centrifugal force is negligible. Mass and bubble radius are very low order against other forces.

$$m_l g - m_g g - \frac{1}{2}C_D\rho_l A U_T^2 + m_g a_{cen} + m^{\bullet}u = ma \tag{40}$$

$$\frac{1}{2}C_D\rho_l A U_T^2 = m_l g + m_g(a_{cen} - g - a) + m^{\bullet}u \tag{41}$$

Fluid contact surface with sphere is supposed to be circular:

$$A = \pi(\frac{d}{2})^2 \tag{42}$$

$$m = \rho V_{bub} \tag{43}$$

Given bubble is also spherical:

$$V_{bub} = \frac{4}{3}\pi(\frac{d}{2})^3 \tag{44}$$

$$\frac{1}{2}C_D\rho_l\pi(\frac{d}{2})^2U_T^2 = \rho_l\frac{4}{3}\pi(\frac{d}{2})^3 g + \rho_g\frac{4}{3}\pi(\frac{d}{2})^3(a_{cen} - g - a) + m^{\bullet}u\frac{\frac{4}{3}\pi(\frac{d}{2})^3}{V_{bub}} \tag{45}$$

After arrangement of 45:

$$C_D = \frac{4}{3}\left[\frac{\rho_l g + \rho_g(a_{cen} - g - a) + \frac{m^{\bullet}u}{V_{bub}}}{\rho_l U_T^2}\right]d_e \tag{46}$$

ρ_g, ρ_l, V_{bubble}, $m^{\bullet}$, u and d_e are liquid and gas density, bubble volume, Mass flow of gas inlet and gas velocity in equivalent outlet and diameter. U_T is the terminal velocity of the bubble $a_{circular}$, a and g are inertia bubble rotational acceleration, and gravity acceleration. Balance form is investigated under steady-state. So, $F_I = ma$ or inertia force is equal to zero. After simplification and ignore the very low amount orders and the forces that are perpendicular to the drag forces. Forces are removed in steady- state and relation 47 is obtained:

$$\frac{1}{2}C_D\rho_l\pi(\frac{d}{2})^2U_T^2 = \rho_l\frac{4}{3}\pi(\frac{d}{2})^3 g - \rho_g\frac{4}{3}\pi(\frac{d}{2})^3 g + m^{\bullet}u\frac{\frac{4}{3}\pi(\frac{d}{2})^3}{V_{bub}} \tag{47}$$

Where [34];

$$C_D = \frac{4}{3}\frac{((\rho_l - \rho_g)g + \frac{m^{\bullet}u}{V_{bub}})d}{\rho_l U_T^2} \tag{48}$$

The drag coefficient is used for Reynolds 0 ≤ Re < 3000. For Reynolds numbers above 3000, the drag coefficient amount is 2.62 [35]. The drag coefficient is obtained external forces precision increasing, significantly. The drag coefficient is analyzed the most relationships diameter precision at low Reynolds.

The drag coefficient in terms of Reynolds number is given as equation 49:

$$C_D = \frac{4}{3}\frac{((\rho_l - \rho_g)g + \frac{m^{\bullet}u}{V_{bubble}})d_e}{\rho_l U_T^2(\frac{\rho_l}{\rho_l}\times\frac{\mu_l^2}{\mu_l^2}\times\frac{d_e^2}{d_e^2})} \tag{49}$$

After arrangement of terminal Reynolds number equivalent parameters in relation:

$$C_D = \frac{4}{3}\frac{((\rho_l - \rho_g)g + \frac{m^{\bullet}u}{V_{bubble}})d_e}{\frac{\rho_l^2U_T^2d_e^2}{\mu_l^2}\times(\frac{1}{\rho_l}\times\frac{\mu_l^2}{1}\times\frac{1}{d_e^2})} \tag{50}$$

Reynolds dimensionless number is [36]:

$$\mathrm{Re}_{Terminal} = \frac{\rho_l U_T d_e}{\mu_l} \tag{51}$$

μ_l is liquid viscosity. Reynolds number in C_D gives:

$$C_D = \frac{4}{3}\frac{((\rho_l-\rho_g)g+\frac{m^{\bullet}u}{V_{bubble}})\rho_l d_e^3}{\mu_l^{\ 2}\ \mathrm{Re}_{Terminal}} \qquad 52$$

4. Conclusion

4.1. Approaches removing bubble diameter in drag coefficient relationship

There are many relations for different towers which can be used according to the shown relationship to remove diameter, in the drag coefficient equations. These relationships are presented the bubble diameter in terms of other mathematical parameters have their own special accuracy. Due to the accuracy and placement in drag coefficient relation, So Drag coefficient can be shown independent of the bubble diameter.

4.1.1.. Bubble mass

One way to calculate the mass is equalization of bubble diameter, its equivalent amount multiplying the density in the bubble volume.

$$m_b = \rho V)_G = \rho_G\frac{4}{3}\pi(\frac{d}{2})^3 \qquad 53$$

Extracting the diameter from 53:

$$d = \sqrt[3]{\frac{6m_b}{\rho\pi}} \qquad 54$$

Bubble volume is obtained from 55:

$$V_{bubble} = \frac{4}{3}\pi(\frac{d}{2})^3 \qquad 55$$

54 is placed in 48:

$$C_D = \frac{4}{3}\frac{((\rho_l-\rho_g)g+\frac{m^{\bullet}u}{V_{bubble}})\sqrt[3]{\frac{6m_b}{\rho_G\pi}}}{\rho_l U_T^2} \qquad 56$$

d is bubble diameter and can be considered as the most accurate diameter. $m^{\bullet}$ is mass flow rate of hole in which bubble is entered to fluid.

One way to calculate the equivalent mass bubble in steady-state conditions is that having bubble density in hand, only bubble volume is necessitated to obtain the mass. Having the gas volume fraction ε_G and the volume of the gas - liquid contact tower V_{Tower}, Total gas volume is obtained. Multiplying tower surface A and its length L, total volume of the tower is obtained. Mass bubble is given with dividing the total gas volume to the number of bubbles, equivalent bubble volume. The number of bubbles in steady-state is obtained via measurement of time needed to increase bubble average and the number of bubbles are separated from hole at the same time. Multiply the numbers of bubbles in the number of hole are given total number of bubbles in steady-state conditions.

$$V_G = \varepsilon_G V_{Tower} \qquad 57$$

$$V_{Tower} = A_T L \qquad 58$$

$$V_{bubble} = \frac{V_{G,total}}{mn} \qquad 59$$

n and m are number of holes and bubbles in tower, respectively.

4.1.2. Mass flow rate

The bubble diameter is calculated using gas mass flow rate. In this method, the equivalent bubble diameter is calculated by mass flow rate of incoming gas of each hole.

$$d = \sqrt{\frac{4m_G^{\bullet}}{\rho_G u_G\pi}} \qquad 60$$

Calculated bubble diameter using mass flow rate is calculated via diameter-independent drag coefficient.

$$C_D = \frac{4}{3}\frac{((\rho_l-\rho_g)g+\frac{m^{\bullet}u_G}{V_{bubble}})\sqrt{\frac{4m_G^{\bullet}}{\rho_G u_G\pi}}}{\rho_l U_T^2} \qquad 61$$

Terminal velocity in the range of bubble diameter larger than 1.5 mm is equal to [37, 39, 44]:

$$U_T = \sqrt{\frac{2\sigma}{d_b\Delta\rho}+\frac{gd_b}{2}} \qquad 62$$

The velocity inside of hole is given as follows:

$$u_G = \frac{m^{\bullet}}{\rho_G A} = \frac{m^{\bullet}}{\rho_G\pi(\frac{d_b}{2})^2} \qquad 63$$

4.1.3. Surface tension and hole diameter

The main equation depending on the diameter is same as relation 64:

$$C_D = \frac{4}{3}\frac{((\rho_l-\rho_g)g+\frac{m^{\bullet}u}{V_{bubble}})d_e}{\rho_l U_T^2} \qquad 64$$

The range of velocity inside of the hole is 1m/s and less. Mass flow rate is measured by 65:

$$m^{\bullet} = \rho_G u_0 A = \rho_G u_0 \pi (\frac{d_0{}^2}{4}) \qquad 65$$

Reynolds number inside of holes is obtained by relation 66:

$$\mathrm{Re}_0 = \frac{\rho_g U_0 d_0}{\mu_g} \qquad 66$$

For very low flow rate of gas:

$$Q_{G0} < [20(\sigma d_0 g_c)^5 /(g\Delta\rho)^2 \rho_L^3]^{1/6} \qquad 67$$

The amount of right side is 0.015718 and values of variables and volumetric flow rate of gas in the left are given in Table 2. So,

Relation 68 is applied to measure bubble diameter in contact with water [39, 43].

$$d_b = (\frac{6d_0\sigma}{\Delta\rho})^{1/3} \qquad 68$$

To calculate terminal velocity, single bubbles is swarmed to the water. For bubble diameter less than 0.7 mm, terminal velocity is given by relation 69 using stockes' law [37].

$$U_T = \frac{g d_b^2 \Delta\rho}{18\mu_L} \qquad 69$$

For bubble diameter larger than 1.4, If the viscosity of the liquid is low, the terminal velocity of single bubbles that are swarmn in water, up to two non-zero digits, is obtained by relation 70 [37, 39, 43, 44]:

$$U_T = \sqrt{\frac{2\sigma}{d_b \Delta\rho} + \frac{g d_b}{2}} \qquad 70$$

$$C_D = \frac{4}{3} \frac{((\rho_l - \rho_g)g + \frac{m^{\bullet}u}{V_{bubble}})(\frac{6d_0\sigma}{\Delta\rho})^{1/3}}{\rho_l U_T^2} \qquad 71$$

$$V_{bubble} = \frac{4}{3}\pi(\frac{d_b}{2})^3 \qquad 72$$

This relationship is true for hole diameter up to 10 mm. For liquids with a viscosity of up to 1000cp [38, 39],

$$d_b = 2.312(\frac{\mu_L Q_{G0}}{\rho_L g})^{1/4} \qquad 73$$

If error percent is measured as $\frac{C_D - C_{D,\mathrm{Real}}}{C_{D,\mathrm{Real}}} \times 100$,

total error rate than the top and bottom of the standard figure are 6.3 approximately.

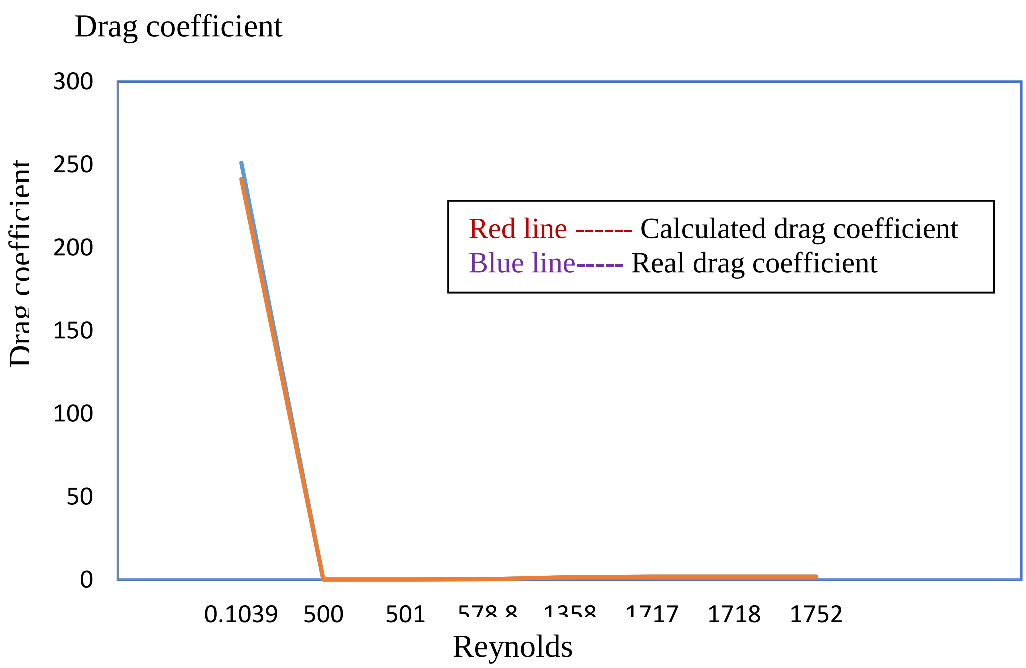

Figure 1; Diagram of Drag Coefficient – Terminal Reynolds

Table.2. It is Shown amount of drag Coefficient independent of the bubble diameter and dependent on surface tension on the terminal Reynolds numbers and other parameters [39].

$d_0\ (m)$	$U_0\ (m/s)$	$m^{\bullet}\ (kg/s)$	Re_0	$Q_{G0} = \frac{m^{\bullet}}{\rho_g},\ (\frac{m^3}{s})$	$d_b\,(m)$	$V_{bub}\ (m^3)$	$V_T\ (m/s)$	Re_T	C_D
.0000080	0.5	1.2158×10^{-10}	0.2521	1.00471×10^{-10}	0.000059	1.074548	0.0018	0.1016	.23251
0.00001	0.5	1.8997×10^{-10}	.31510	1.57×10^{-10}	0.001648	2.3417×10^{-9}	0.31	500	0.222
0.00001	1	9.4985×10^{-11}	.63030	7.85×10^{-11}	0.001648	2.3417×10^{-9}	0.31	501	0.222
0.00005	0.5	4.74925×10^{-9}	1.5758	3.925×10^{-9}	0.001953	3.894619×10^{-9}	0.29	575.8	0.3029
0.0005	0.2	1.8997×10^{-7}	6.3557	1.57×10^{-7}	0.006037	1.1511×10^{-7}	0.23	1358	1.488
0.001	0.05	1.8997×10^{-7}	3.1778	1.57×10^{-7}	0.007634	2.3295×10^{-7}	0.23	1717.9	1.883
0.001	0.1	4.2641×10^{-7}	7	4.3504×10^{-7}	0.007636	2.3295×10^{-7}	0.23	1718	1.8832
0.001	0.055	2.2×10^{-7}	3.4955	1.8181×10^{-7}	0.007636	2.33×10^{-7}	0.23	1752	1.881
0.001	0.2	0.000076	127.11	6.3×10^{-5}	0.016439	0.000004	0.28	4621	2.5981

Table. 3. Property of water filtered and air in 19 ^{0}c [35].

ρ_g	$1.21 Kg/m^3$
ρ_l	$998 Kg/m^3$
μ_l	$0.00102 Kg/ms$
σ_l	$0.0729 N/m$

Table, 4. Amount of Error Percent in different Reynolds in addition to the real drag coefficient and calculated is shown [35].

Terminal Reynolds Re_T	calculated drag coefficient C_D	Real drag coefficient $C_{D,\mathrm{Real}}$	Error Percent $(\frac{C_D - C_{D,\mathrm{Real}}}{C_{D,\mathrm{Real}}} \times 100)$
0.1039	251	241.23	+4
500	0.222	0.218	+1.8
501	0.222	0.2185	+1.6
578.8	0.3029	0.295	+2.6
1358	1.488	1.5	-0.8
1717	1.883	1.92	-2
1718	1.883	1.928	-2.33
1752	1.881	1.928	-2.43

5. Conclusion

The drag coefficients are investigated depended on the size and configuration of bubbles. The drag coefficient of Krishna et al is used dependent on bubble diameter. Schiller - Nauman model drag coefficient is estimated with 9% error and dependence on bubble diameter. In this article, the modern drag coefficients are studied independent on the diameter and shape of the bubble.
The Drag coefficients are resulted theoretical, mathematical and experimental independent and dependent of diameter bubble. The new Drag coefficient is presented dependent on surface tension and diameter of the tower hole with 6.3 of error approximately.

References

[1] R. Rahimi, S. Hajghani, Investigation of Heat Transfer Parameters of a Bundle of Heaters in a Simple Bubble Column Reactor Using CFD Method, Chemical and Petroleum Engineering 49 (2015) 31-49.
[2] Allen, M. S., 1900. Philos. Mag, 50.pp. 323-338, 519-534.
[3] Langmuir, I., and Blodgett, K. B., 1948 C.S. Army Air Force Tech. Rep. No. 5418.
[4] Dalla, Ville, J.M. 1948 Micrometrics, Pitman Publishing Co. New York.
[5] Gilbert, M. Davies, L., and Altman, D.,1955 Jet Propul,. 25, pp. 26-30.
[6] Moore. D. W., J. 1963 Fluid Mech. 16. Pp. 161 -176. [7] Kurten.H., Raasch, J., and Rumpf, H., Chem - Ing - Tech. 38. (1966). pp. 941-948.
[8] Abraham. F. F.1970 Phys.Fluids. 13. Pp. 2194-2195.
[9] Clift, R, and Gauvin, W. H., 1970 Proc. Chenleca 70, 1, pp. 14 28.
[10] Tanaka, Z., and Iinoya, K., J. 1970 Chem. Eng. Jpn, 3. pp. 261-262.
[11] Ihme, F. Schmidt - Traub, H., and Brauer, H. 1972. Chern-Ing - Teclt, 44. , pp. 306 - 313.
[12] Brauer, H., and Mewes, D., 1972 Chem,-Ing.-Tech. 44. pp. 865-868.
[13] White, F.M. 1974. Viscous fluid flow, McGraw-Hill, New York .
[14] D. Ma, G.J., Ahmadi, A thermo dynamical formulation for dispersed turbulent flows, 1: basic theory, Int. J. Multiphase Flow 16 (1990) 323–340.
[15] S. Grevskott, B.H. Sannaes, M.P. Dudkovic, K.W. Hjarbo, H.F. Svendsen, Liquid circulation, bubble size distributions and solids movement in two and three-phase bubble columns, Chem. Eng. Sci 51 (1996) 1703–1713.
[16] K. Tsuchiya, A. Furumoto, L.S. Fan, J., Zhang, Suspension viscosity and bubble size velocity in liquid–solid fluidized beds, Chem. Eng. Sci 52 (1997) 3053–3066.
[17] G.L., Lane, M.P., Schwarz, G.M., Evans, Modelling of the interaction between gas and liquid in stirred vessels. In: Proceedings of 10th European Conference on Mixing, Delft, The Netherlands (2000) 197-204.
[18] A. Tomiyama, I. Kataoka, I. Zˇun, T. Sakaguchi, Drag coefficients of single bubbles under normal and micro gravity conditions, JSME Int. J. 42 (1998) 472 – 479.
[19] Tomiyama, A. drag lift and virtual mass forces acting on a single bubble, Third International Symposium on Two-Phase Flow Modeling and experimentation, 2004.
[20] B. Hameed. A.N. Mahooda, A,O Campbell, RB. Thorpe sharif, Heat transfer measurement in a three-phase direct-contact condenser under flooding conditions, applied Thermal Engineering 95 (2016) 106–114.
[21] L.A., Schiller, Z.Naumaan, A drag coefficient correlation Ver Dtsch, Ing. 77. (1935) 138.
[22] M. Ishii, N. Zuber, Drag coefficient and relative velocity in bubbly, droplet or particulate flows, AIChE J. 25. (1979) 843–855.
[23] Krishna, Van Baten, J., Ellenberger, A.P., Higler, and Taylor, CFD Simulations of Sieve Tray Hydrodynamics, Chemical Engineering Research and Design. 77. (1999) 639–646.
[24] D.L., Bennett, R. Agrawal, and P.J., Cook, New pressure drop correlation for sieve tray distillation columns, AIChE J, 29, (1983) 434 - 442.
[25] D. Noriler, H.F., Meier, A.A.C. Wolf, Barros , M.R. Maciel, Thermal fluid dynamics analysis of gas–liquid flow on a distillation sieve tray, Chemical Engineering Journal, Volume 136, (2008) 133–143.
[26] L. Zhang, Ch Yang,.And Z. Sh. Mao, Numerical simulation of a bubble rising in shear-thinning fluids, Journal of Non-Newtonian Fluid Mechanics, 165. (2015) 555–567.
[27] R. Byron Bird, Warren E. Stewart ,Edwin N. Lightfoot, Transport Phenomena, John Wiley & Sons, Inc, Second Edition. 1924. pp. 80-90.
[28] A. Brucato, F., Grisafi, G., Montante, Particle drag coefficient in turbulent fluids. Chemical Engineering Science 53, (1998) 3295 - 3314.
[29] J.R Grace, T. Wairegi, T.H., Nguyen, Shapes and velocities of single drops and bubbles moving freely through immiscible liquids, Trans. Inst. Chem. Eng 54 (1976) 167–173.
[30] Fengling Yang , Shenjie Zhou , Xiaohui An , Gas–liquid hydrodynamics in a vessel stirred by dual dislocated-blade Rushton impellers, Chinese Journal of Chemical Engineering 23 (2015) 1746 -1754.
[31] Ning Yang, Zongying Wu, Jianhua Chen, Yuhua Wang, Jinghai Li, Multi-scale analysis of gas-liquid interaction and CFD simulation of gas–liquid flow in bubble columns, Chemical Engineering Science 66 (2011) 3212–3222.
[32] C. E., Lapple, Particle Dynamics. Eng. Res. Lab. E.I .1951, DuPont de Nemours and Co.Wilmington. Delawarc.
[33] J. M., Kendall. reported by Kiichemann. D. J., FIuid Mech. 21, 1965 pp. 1 - 20.
[34] Halliday, Resnick, Fundamentals of Physics, Tenth Edition, WILy 2014 pp. 114-115.
[35] W.L., Habermn sad, R.K. Morton, Armed Servic'es Technical Information gency AD 19377, Navy a Department The David W. Taylor, W. Taylor Model Basin 1953 pp.1- 48.
[36] B R., Munson, D. F. Young, T. H., Okiishi, W. W., Huebsch, Fundamentals of Fluid Mechanics, John Wiley & Sons, Inc. Seventh Edition, USA, 2015 pp. 677-679.
[37] H. D., Mendelson, AIChE J., 13., 250.1967.
[38] J. F., Davidson, and B. G. Shuler,.Trans. I. Inst, Chem. Eng, lond, 38. 144. 1960.
[39] R. E., Treybal, MASS-TRANSFER OPERATION, Third Edition, International Edition. pp. 136-216. 1981.
[40] R., Clift, J. R., Grace, M. E., Weber, Bubbles, Drops, and Particles, Journal of Academic Press. New York 10003. 1978 pp. 3-14.
[41] D., Law. F. Battaglia, T.J., Heindel, Model validation for lowand high superficial gas velocity bubble column flows, Chemical Engineering Scince 63 (2008). 4605–4616.
[42] M.V. Tabib, S.A. Roy, J.B. Joshi, CFD simulation of bubble column an analysis of interphase forces and

turbulence models, Chemical Engineering 139 (2008) 589–614.
[43] W, Van krevelen, and P, J, Hoftijzer: Chemical Engineering, prog, 46 (1950). 9.
[44] C,C., Maneri; and H, D Mendelson: AIChE, J., 14, 295 (1968).

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